



# Decomposition and stability of linear singularly perturbed systems with two small parameters

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In the domain  $\Omega = \{(t, \varepsilon_1, \varepsilon_2) : t \in \mathbb{R}, \varepsilon_1 > 0, \varepsilon_2 > 0\}$ , we consider a linear singularly perturbed system with two small parameters

$$\begin{cases} \dot{x}_0 = A_{00}x_0 + A_{01}x_1 + A_{02}x_2, \\ \varepsilon_1 \dot{x}_1 = A_{10}x_0 + A_{11}x_1 + A_{12}x_2, \\ \varepsilon_1 \varepsilon_2 \dot{x}_2 = A_{20}x_0 + A_{21}x_1 + A_{22}x_2, \end{cases}$$

where  $x_0 \in \mathbb{R}^{n_0}$ ,  $x_1 \in \mathbb{R}^{n_1}$ ,  $x_2 \in \mathbb{R}^{n_2}$ . In this paper, schemes of decomposition and splitting of the system into independent subsystems by using the integral manifolds method of fast and slow variables are investigated. We give the conditions under which the reduction principle is truthful to study the stability of zero solution of the original system.

*Key words and phrases:* singularly perturbed system, decomposition, splitting, stability, integral manifold.

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## Introduction

The theory of singularly perturbed systems of differential equations has been intensively developed recently, and its methods have been widely used for the study of various problems of science and technique. Singularly perturbed models describe dynamic processes in physics, chemical kinetics, theory of automatic control, hydroscopic systems, economical and biological processes etc.

For singularly perturbed systems, an important task is to develop asymptotic methods for investigating the qualitative behavior of both individual solutions and their sets. Effective method of investigation of singularly perturbed systems is integral manifold method [4, 11], which allows to reduce the dimension of the initial system on the integral manifold. In many applications of singularly perturbed systems an important role is played by the transformation of variables, which makes it possible to decompose the initial system to a block-triangular form [7]. In particular, algorithms for constructing asymptotic decompositions of an integral manifold by degrees of a small parameter, that can be effectively implemented in systems of computer algebra, are investigated in [3, 6].

The application of the integral manifold method to the decomposition and splitting of linear singularly perturbed systems into independent fast and slow subsystems proved to be

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interesting and effective [2, 9, 10]. In particular, for a linear system

$$\begin{cases} \dot{x} = A_{11}x + A_{12}y, \\ \varepsilon\dot{y} = A_{21}x + A_{22}y \end{cases}$$

were established the conditions for existence of variables substitution

$$\begin{cases} x = u + \varepsilon P(t, \varepsilon)v, \\ y = v + H(t, \varepsilon)x, \end{cases}$$

which transforms the initial system into two independent subsystems

$$\begin{aligned} \dot{u} &= (A_{11} + A_{12}H)u, \\ \varepsilon\dot{v} &= (A_{22} - \varepsilon HA_{12})v. \end{aligned}$$

The application of the method of integral manifolds of fast and slow variables to the studying of linear singularly perturbed systems with many small parameters was carried out in [1, 5, 8, 12]. In this paper, we establish the reduction principle for investigation the stability of linear singularly perturbed systems with two small parameters.

## 1 Decomposition

In the domain  $\Omega = \{(t, \varepsilon_1, \varepsilon_2) : t \in \mathbb{R}, \varepsilon_1 > 0, \varepsilon_2 > 0\}$ , we consider a linear singularly perturbed system

$$\begin{cases} \dot{x}_0 = A_{00}x_0 + A_{01}x_1 + A_{02}x_2, \\ \varepsilon_1\dot{x}_1 = A_{10}x_0 + A_{11}x_1 + A_{12}x_2, \\ \varepsilon_1\varepsilon_2\dot{x}_2 = A_{20}x_0 + A_{21}x_1 + A_{22}x_2, \end{cases} \quad (1)$$

where  $x_0 \in \mathbb{R}^{n_0}$ ,  $x_1 \in \mathbb{R}^{n_1}$ ,  $x_2 \in \mathbb{R}^{n_2}$ ,  $A_{ij} = A_{ij}(t)$ ,  $i, j = \overline{0, 2}$ , are  $n_i \times n_j$  matrices, and  $\varepsilon_1, \varepsilon_2$  are small positive parameters.

Let the following conditions hold:

C.1) matrices  $A_{ij}(t)$ ,  $i, j = \overline{0, 2}$ , are uniformly bounded in  $t \in \mathbb{R}$  by a positive constant  $M$ ;

C.2) eigenvalues  $\lambda_i = \lambda_i(t)$ ,  $i = \overline{1, n_2}$ , of the matrix  $A_{22}(t)$  satisfy the inequality

$$\operatorname{Re} \lambda_i \leq -2\beta < 0.$$

Let us make in (1) the substitution

$$x_0 = y_0 + \varepsilon_1\varepsilon_2 H_0 w, \quad x_1 = y_1 + \varepsilon_2 H_1 w, \quad x_2 = w + P_0 x_0 + P_1 x_1, \quad (2)$$

where  $H_0, H_1, P_0, P_1$  are matrix functions of appropriate dimensions.

If matrices  $P_0$  and  $P_1$  are solutions of the system

$$\begin{cases} \varepsilon_1\varepsilon_2\dot{P}_0 = A_{20} + A_{22}P_0 - \varepsilon_1\varepsilon_2 P_0 A_{00} - \varepsilon_1\varepsilon_2 P_0 A_{02}P_0 - \varepsilon_2 P_1 A_{10} - \varepsilon_2 P_1 A_{12}P_0, \\ \varepsilon_1\varepsilon_2\dot{P}_1 = A_{21} + A_{22}P_1 - \varepsilon_1\varepsilon_2 P_0 A_{01} - \varepsilon_1\varepsilon_2 P_0 A_{02}P_1 - \varepsilon_2 P_1 A_{11} - \varepsilon_2 P_1 A_{12}P_1, \end{cases} \quad (3)$$

then system (1) will have the following form

$$\begin{cases} \dot{y}_0 = (A_{00} + A_{02}P_0)y_0 + (A_{01} + A_{02}P_1)y_1 + (\varepsilon_1\varepsilon_2 A_{00}H_0 + \varepsilon_2 A_{01}H_1 - \varepsilon_1\varepsilon_2 \dot{H}_0 \\ \quad + A_{02}(E + \varepsilon_1\varepsilon_2 P_0 H_0 + \varepsilon_2 P_1 H_1) - H_0(A_{22} - \varepsilon_1\varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}))w, \\ \varepsilon_1\dot{y}_1 = (A_{10} + A_{12}P_0)y_0 + (A_{11} + A_{12}P_1)y_1 + (\varepsilon_1\varepsilon_2 A_{10}H_0 + \varepsilon_2 A_{11}H_1 - \varepsilon_1\varepsilon_2 \dot{H}_1 \\ \quad + A_{12}(E + \varepsilon_1\varepsilon_2 P_0 H_0 + \varepsilon_2 P_1 H_1) - H_1(A_{22} - \varepsilon_1\varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}))w, \\ \varepsilon_1\varepsilon_2\dot{w} = (A_{22} - \varepsilon_1\varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12})w. \end{cases} \quad (4)$$

If matrices  $H_0$  and  $H_1$  are solutions of the system

$$\begin{cases} \varepsilon_1 \varepsilon_2 \dot{H}_0 = \varepsilon_1 \varepsilon_2 A_{00} H_0 + A_{02} (E + \varepsilon_1 \varepsilon_2 P_0 H_0 + \varepsilon_2 P_1 H_1) \\ \quad - H_0 (A_{22} - \varepsilon_1 \varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}) + \varepsilon_2 A_{01} H_1, \\ \varepsilon_1 \varepsilon_2 \dot{H}_1 = \varepsilon_1 \varepsilon_2 A_{10} H_0 + A_{12} (E + \varepsilon_1 \varepsilon_2 P_0 H_0 + \varepsilon_2 P_1 H_1) \\ \quad - H_1 (A_{22} - \varepsilon_1 \varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}) + \varepsilon_2 A_{11} H_1, \end{cases} \quad (5)$$

then system (4) will have the form

$$\begin{cases} \dot{y}_0 = B_{00} y_0 + B_{01} y_1, \\ \varepsilon_1 \dot{y}_1 = B_{10} y_0 + B_{11} y_1, \\ \varepsilon_1 \varepsilon_2 \dot{w} = B_{22} w, \end{cases} \quad (6)$$

where  $B_{ij} = A_{ij} + A_{i2} P_j$ ,  $i, j = 0, 1$ ,  $B_{22} = A_{22} - \varepsilon_1 \varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}$ .

Under conditions C.1), C.2) and a sufficiently small value of the small parameter  $\varepsilon_2 > 0$ , in [8] it is established the existence of bounded solutions of systems (3) and (5) by which system (1) is reduced to the form (6).

Suppose that for the system (1) we should solve a Cauchy problem with initial conditions  $(t_0, x_{00}, x_{10}, x_{20})$ . Let us show that there is a solution of the system (6) with initial conditions  $(t_0, y_{00}, y_{10}, w_0)$  for which the equalities (2) are true. By the uniqueness of the solution, it suffices to show that (2) holds for  $t = t_0$ , that is

$$\begin{aligned} x_{00} &= y_{00} + \varepsilon_1 \varepsilon_2 H_0^0 w_0, \\ x_{10} &= y_{10} + \varepsilon_2 H_1^0 w_0, \\ x_{20} &= w_0 + P_0^0 x_{00} + P_1^0 x_{10}. \end{aligned} \quad (7)$$

From (7) we can get

$$\begin{aligned} w_0 &= x_{20} - P_0^0 x_{00} - P_1^0 x_{10}, \\ y_{10} &= x_{10} - \varepsilon_2 H_1^0 (x_{20} - P_0^0 x_{00} - P_1^0 x_{10}), \\ y_{00} &= x_{00} - \varepsilon_1 \varepsilon_2 H_0^0 (x_{20} - P_0^0 x_{00} - P_1^0 x_{10}). \end{aligned} \quad (8)$$

Therefore, each solution of the system (1) can be represented as (2), where  $y_0, y_1, w$  is the solution of the system (6) with initial conditions (8).

Let us present this solution as

$$\begin{aligned} x_0 &= x_0(t, \varepsilon_1, \varepsilon_2) = y_0(t, \varepsilon_1, \varepsilon_2) + \varphi_1(t, \varepsilon_1, \varepsilon_2), \\ x_1 &= x_1(t, \varepsilon_1, \varepsilon_2) = y_1(t, \varepsilon_1, \varepsilon_2) + \varphi_2(t, \varepsilon_1, \varepsilon_2), \\ x_2 &= x_2(t, \varepsilon_1, \varepsilon_2) = P_0 y_0(t, \varepsilon_1, \varepsilon_2) + P_1 y_1(t, \varepsilon_1, \varepsilon_2) + \varphi_3(t, \varepsilon_1, \varepsilon_2), \end{aligned} \quad (9)$$

where

$$\varphi_1(t, \varepsilon_1, \varepsilon_2) = \varepsilon_1 \varepsilon_2 H_0^0 w_0, \varphi_2(t, \varepsilon_1, \varepsilon_2) = \varepsilon_2 H_1^0 w_0, \varphi_3(t, \varepsilon_1, \varepsilon_2) = w_0 + P_0 \varphi_1(t, \varepsilon_1, \varepsilon_2) + P_1 \varphi_2(t, \varepsilon_1, \varepsilon_2).$$

## 2 Splitting and reduction principle

Denote  $Q(t, s, \varepsilon_1, \varepsilon_2)$  the fundamental matrix of the equation  $\varepsilon_1 \varepsilon_2 \dot{x}_2 = A_{22} x_2$ .

The uniform boundedness of the matrix  $A_{22}$  in the domain  $\Omega$  and condition C.2) provide the estimate (see [9, 11, 12])

$$\|Q(t, s, \varepsilon_1, \varepsilon_2)\| \leq K e^{-\frac{3\beta}{2\varepsilon_1 \varepsilon_2}(t-s)} \quad (10)$$

for some  $K > 0$  and any  $-\infty < s \leq t < \infty$ .

**Lemma 1.** *Let conditions C.1), C.2) be true. Then there exists  $\bar{\varepsilon}_2$  such that for  $0 < \varepsilon_2 \leq \bar{\varepsilon}_2$  the fundamental matrix  $\bar{Q}(t, s, \varepsilon_1, \varepsilon_2)$  of the equation*

$$\varepsilon_1 \varepsilon_2 \dot{w} = (A_{22} - \varepsilon_1 \varepsilon_2 P_0 A_{02} - \varepsilon_2 P_1 A_{12}) w \quad (11)$$

satisfies the estimate

$$\left| \bar{Q}(t, s, \varepsilon_1, \varepsilon_2) \right| \leq K e^{-\frac{\beta}{\varepsilon_1 \varepsilon_2}(t-s)}. \quad (12)$$

*Proof.* Rewrite the equation (11) as

$$\varepsilon_1 \varepsilon_2 \dot{w} = A_{22} w - (\varepsilon_1 \varepsilon_2 P_0 A_{02} + \varepsilon_2 P_1 A_{12}) w.$$

The fundamental matrix  $\bar{Q}(t, s, \varepsilon_1, \varepsilon_2)$  satisfies the integral equation

$$\bar{Q} = Q - \int_{t_0}^t Q (\varepsilon_1 \varepsilon_2 P_0 A_{02} + \varepsilon_2 P_1 A_{12}) \bar{Q} ds.$$

Using estimation (10), condition C.1) and inequalities for matrices  $\|P_0\| \leq \frac{KM}{\beta}$ ,  $\|P_1\| \leq \frac{KM}{\beta}$  (see [8]), we have

$$\left| \bar{Q} \right| \leq K e^{-\frac{3\beta}{2\varepsilon_1 \varepsilon_2}(t-s)} + \int_{t_0}^t \frac{K^2 M^2}{\beta} (\varepsilon_1 \varepsilon_2 + \varepsilon_2) \left| \bar{Q} \right| e^{-\frac{3\beta(t-s)}{2\varepsilon_1 \varepsilon_2}} ds.$$

Applying the Gronwall-Bellman inequality, we obtain

$$\left| \bar{Q} \right| e^{\frac{3\beta t}{2\varepsilon_1 \varepsilon_2}} \leq K_1 e^{\int_{t_0}^t \frac{K^2 M^2}{\beta} (\varepsilon_1 \varepsilon_2 + \varepsilon_2) ds} = K_1 e^{\frac{K^2 M^2}{\beta} (\varepsilon_1 \varepsilon_2 + \varepsilon_2)(t-t_0)}.$$

Therefore,  $\left| \bar{Q} \right| \leq K e^{\left( \frac{K^2 M^2}{\beta} (\varepsilon_1 \varepsilon_2 + \varepsilon_2) - \frac{3\beta}{2\varepsilon_1 \varepsilon_2} \right) t}$ . The last inequality for  $\varepsilon < \frac{\beta}{\sqrt{2\varepsilon_2(\varepsilon_1+1)KM}} = \bar{\varepsilon}_2$  takes the form

$$\left| \bar{Q} \right| \leq K e^{-\frac{\beta}{\varepsilon_1 \varepsilon_2}(t-s)}.$$

□

From the boundedness of the matrices  $H_0, H_1$  and the estimate (12) we obtain that there is such positive number  $N$  that for  $t \geq t_0$  the following inequalities hold

$$\begin{aligned} |\varphi_1(t, \varepsilon_1, \varepsilon_2)| &\leq \varepsilon_1 \varepsilon_2 N e^{-\frac{\beta}{\varepsilon_1 \varepsilon_2}(t-t_0)} |w_0|, \\ |\varphi_2(t, \varepsilon_1, \varepsilon_2)| &\leq \varepsilon_2 N e^{-\frac{\beta}{\varepsilon_1 \varepsilon_2}(t-t_0)} |w_0|, \\ |\varphi_3(t, \varepsilon_1, \varepsilon_2)| &\leq N e^{-\frac{\beta}{\varepsilon_1 \varepsilon_2}(t-t_0)} |w_0|. \end{aligned} \quad (13)$$

From the obtained relations (9) and estimates (13) we obtain that the system (1) has the integral manifold

$$x_2 = P_0 x_0 + P_1 x_1,$$

the motion on which describes the subsystem of the first two equations of the system (6)

$$\begin{cases} \dot{y}_0 = B_{00} y_0 + B_{01} y_1, \\ \varepsilon_1 \dot{y}_1 = B_{10} y_0 + B_{11} y_1. \end{cases} \quad (14)$$

Any solution of system (1) is represented as the sum of some solution that lies on the integral manifold and exponentially decreasing terms. From the representation (9) and the estimates (13) we obtain statements for the investigation of the stability of the zero solution of system (1).

**Theorem 1.** *Let the conditions C.1), C.2) be true. Then for sufficiently small  $\varepsilon_1, \varepsilon_2$  the zero solution of system (1) is stable (asymptotically stable, unstable) if and only if the zero solution of subsystem (14) is stable (asymptotically stable, unstable).*

Let us now consider system (6), which has already been studied in [9,10]. Let the following condition be true

C.3) eigenvalues  $\lambda_i = \lambda_i(t)$ ,  $i = \overline{0, n_1}$ , of the matrix  $B_{11}(t, \varepsilon_1, \varepsilon_2)$  satisfy the inequality

$$\operatorname{Re} \lambda_i \leq -2\gamma < 0.$$

Then there exists  $\varepsilon_1^* > 0$  such that for  $0 < \varepsilon_1 < \varepsilon_1^*$  the substitution of variables (see [8,9])

$$\begin{cases} y_0 = u + \varepsilon_1 H(t, \varepsilon_1, \varepsilon_2)v, \\ y_1 = v + P(t, \varepsilon_1, \varepsilon_2)y_0 \end{cases} \quad (15)$$

splits the system (6) into independent subsystems

$$\begin{cases} \dot{u} = (B_{00} + B_{01})Hu, \\ \varepsilon_1 \dot{v} = (B_{11} - \varepsilon_1 HB_{01})v. \end{cases} \quad (16)$$

The matrix functions  $P$  and  $H$  are uniformly bounded solutions of such equations

$$\begin{cases} \varepsilon_1 \dot{P} = \varepsilon_1 (B_{00} + B_{01}H)P + B_{01} - P(B_{11} - \varepsilon_1 HB_{01}), \\ \varepsilon_1 \dot{H} = B_{10} + B_{11}H - \varepsilon_1 H(B_{00} - B_{01}H). \end{cases}$$

By expressing the old variables  $x_0, x_1, x_2$  through the new ones  $u, v, w$  we obtain the following result.

**Theorem 2 ([8]).** *Let conditions C.1), C.2), C.3) be satisfied. Then for sufficiently small values  $\varepsilon_1, \varepsilon_2$ , there is a nondegenerate substitution of variables*

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} E & \varepsilon_1 H & \varepsilon_1 \varepsilon_2 H_0 \\ P & E + \varepsilon_1 PH & \varepsilon_2 H_1 \\ P_0 + P_1 P & P_1 + \varepsilon_1 (P_0 + P_1 P) H & E + \varepsilon_1 \varepsilon_2 P_0 H_0 + \varepsilon_2 P_1 H_1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix},$$

by which system (1) is transformed to three independent subsystems

$$\begin{cases} \dot{u} = (B_{00} + B_{01}H)u, \\ \varepsilon_1 \dot{v} = (B_{11} - \varepsilon_1 HB_{01})v, \\ \varepsilon_1 \varepsilon_2 \dot{w} = B_{22}w. \end{cases}$$

In system (16) the first equation describes the behavior of system (6) on the invariant manifold of slow variables, and the second equation provides the behavior of system (6) on the invariant manifold of fast variables.

From representation (15) under condition C.3), it was established in [9,10] that the stability of system (6) is equivalent to the stability of the first equation of system (16).

Let us summarize these considerations in the form of reduction principle for investigating the stability of the zero solution of the original system (1).

**Theorem 3.** *Let conditions C.1), C.2), C.3) be satisfied. Then for sufficiently small values  $\varepsilon_1, \varepsilon_2$  the zero solution of system (1) is stable (asymptotically stable, unstable) if and only if the zero solution of equation*

$$u'(t) = (B_{00} + B_{01}H)u$$

*is stable (asymptotically stable, unstable).*

**Example.** As an example of application of construction of splitting transformation and application of the principle of reduction we will consider system of singularly perturbed differential equations

$$\begin{cases} \dot{x}_0 = x_0 + x_1 + x_2, \\ \varepsilon_1 \dot{x}_1 = x_0 + x_1 + x_2, \\ \varepsilon_1 \varepsilon_2 \dot{x}_2 = -x_0 - x_1 - x_2. \end{cases} \quad (17)$$

Let us make in (17) the substitution

$$x_0 = y_0 + \varepsilon_1 \varepsilon_2 h_0 w, \quad x_1 = y_1 + \varepsilon_2 h_1 w, \quad x_2 = w + p_0 x_0 + p_1 x_1. \quad (18)$$

To find the coefficients  $p_0, p_1, h_0, h_1$ , taking into account the stationarity and form of the system (17) from relations (3) and (5) we obtain the following systems of equations:

$$\begin{cases} -1 - p_0 - \varepsilon_1 \varepsilon_2 p_0 - \varepsilon_1 \varepsilon_2 p_0^2 - \varepsilon_2 p_1 - \varepsilon_2 p_0 p_1 = 0, \\ -1 - p_1 - \varepsilon_1 \varepsilon_2 p_0 - \varepsilon_1 \varepsilon_2 p_0 p_1 - \varepsilon_2 p_1 - \varepsilon_2 p_1^2 = 0, \\ h_0(-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2) = 1, \\ h_1(-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2) = 1. \end{cases}$$

Finding the bounded solutions of these systems, we get

$$p_0 = p_1 = -1, \quad h_0 = h_1 = \frac{1}{-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2}.$$

Thus, the substitution of variables (18) takes the form

$$x_0 = y_0 + \frac{\varepsilon_1 \varepsilon_2}{-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2} w, \quad x_1 = y_1 + \frac{\varepsilon_2}{-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2} w, \quad x_2 = w - x_0 - x_1.$$

In this case, the system (17) is split into the system of independent equations

$$\begin{cases} \dot{y}_0 = 0, \\ \varepsilon_1 \dot{y}_1 = 0, \\ \varepsilon_1 \varepsilon_2 \dot{w} = (-1 + \varepsilon_1 \varepsilon_2 + \varepsilon_2)w. \end{cases} \quad (19)$$

According to Theorem 3, the stability of the zero solution of system (17) is equivalent to the stability of the zero solution of the first equation of system (19). Thus, based on the principle of reduction, we can conclude that the zero solution of system (17) is stable, but not asymptotic.

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В області  $\Omega = \{(t, \varepsilon_1, \varepsilon_2) : t \in \mathbb{R}, \varepsilon_1 > 0, \varepsilon_2 > 0\}$  досліджується лінійна сингулярно збурена система з двома малими параметрами

$$\begin{cases} \dot{x}_0 = A_{00}x_0 + A_{01}x_1 + A_{02}x_2, \\ \varepsilon_1 \dot{x}_1 = A_{10}x_0 + A_{11}x_1 + A_{12}x_2, \\ \varepsilon_1 \varepsilon_2 \dot{x}_2 = A_{20}x_0 + A_{21}x_1 + A_{22}x_2, \end{cases}$$

де  $x_0 \in \mathbb{R}^{n_0}$ ,  $x_1 \in \mathbb{R}^{n_1}$ ,  $x_2 \in \mathbb{R}^{n_2}$ . Розглянуто схеми декомпозиції та розщеплення системи на незалежні підсистеми за допомогою інтегральних многовидів швидких та повільних змінних. Встановлено умови, при виконанні яких справедливий принцип зведення для дослідження стійкості нульового розв'язку вихідної системи.

*Ключові слова і фрази:* сингулярно збурена система, декомпозиція, розщеплення, стійкість, інтегральний многовид.