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k-BITRANSITIVE AND COMPOUND OPERATORS ON BANACH SPACES

In this this paper, we introduce new classes of operators in complex Banach spaces, which we call k-bitransitive operators and compound operators to study the direct sum of diskcyclic operators. We create a set of sufficient conditions for an operator to be k-bitransitive or compound. We give a relation between topologically mixing operators and compound operators. Also, we extend the Godefroy-Shapiro Criterion for topologically mixing operators to compound operators.

Key words and phrases: hypercyclic operators, diskcyclic operators, weakly mixing operators, direct sums.

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INTRODUCTION

A bounded linear operator T on a separable Banach space X is hypercyclic if there is a vector $x \in X$ such that $Orb(T,x) = \{T^n x : n \geq 0\}$ is dense in X, such a vector x is called hypercyclic for T. Similarly, an operator T is called diskcyclic if there is a vector $x \in X$ such that the disk orbit $\mathbb{D}Orb(T,x) = \{\alpha T^n x : \alpha \in \mathbb{C}, |\alpha| \leq 1, n \in \mathbb{N}\}$ is dense in X, such a vector x is called diskcyclic for T. In Banach spaces, hypercyclic (or diskcyclic) operators are identical to topological transitive (or disk transitive, respectively) [3, 4].

Definition 1. *A bounded linear operator $T : X \to X$ is called*

1. *topological transitive, if for any two non empty open sets U and V, there exists a positive integer n such that $T^n U \cap V \neq \varnothing$;*
2. *disk transitive, if for any two non empty open sets U and V, there exist a positive integer n and $\alpha \in \mathbb{C}, 0 < |\alpha| \leq 1$, such that $T^n \alpha U \cap V \neq \varnothing$.*

For more information on hypercyclic and diskcyclic operators the reader may refer to [2, 3, 4, 11].

A sufficient condition for hypercyclicity, the well known Hypercyclicity Criterion, independently discovered by Kitai [13] and Gethner and Shapiro [9]. Latter on, Godefroy and Shapiro [10] created another hypercyclic criterion which is called Godefroy-Shapiro Criterion, that is a set of sufficient condition in terms of the eigenvalues of an operator to be hypercyclic.

In 1982, Kitai [13] showed that if $T_1 \oplus T_2$ is hypercyclic, then T_1 and T_2 are hypercyclic. However, for the converse, Salas constructed an operator T such that both it and its adjoint T^* are hypercyclic, and so that their direct sum $T \oplus T^*$ is not. Moreover, Herrero asked in [12]

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whether $T \oplus T$ is hypercyclic whenever T is. De la Rosa and Read [7] showed that the Herrero's question is not true by giving a hypercyclic operator T such that $T \oplus T$ is not. On the other hand, if T satisfies hypercyclic criterion, then $T \oplus T$ is hypercyclic [4]. In 1999, Bés and Peris [5] proved that the converse is also true; that is, if $T \oplus T$ is hypercyclic, then T satisfies hypercyclic criterion.

For diskcyclic operators, Zeana proved that if the direct sum of k operators is diskcyclic then every operator is diskcyclic [14]. However, the converse is unknown. Particularly, we have the following question:

Question 1. *If there are k diskcyclic operators, what about their direct sum?*

The main purpose of this paper is to give a partial answer to this question by defining and studying a new class of operators, namely k-bitransitive operators. We determine conditions that ensure a linear operator to be k- bitransitive which is called k-bitransitive criterion. We use this criterion to show that in some cases the direct sum of k diskcyclic operators is k-bitransitive. Then, we define compound operators as a general form of mixing operators [6] to show that under certain conditions the direct sum of k diskcyclic operators is k-bitransitive. Then, we studied some properties of compound operators. In particular, we give some sufficient conditions for an operator to be compound which is refer to compound criterion. We use this criterion to show that not every compound operator is mixing. Finally, we extend Godefroy-Shapiro Criterion [1, Theorem 1.3] for mixing operators to compound operators. In particular, a special case of Theorem 3 is when $p = 1$ which is Godefroy-Shapiro Criterion.

1 MAIN RESULTS

In this this paper, all Banach spaces are separable over the field $\mathbb{C}$ of complex numbers. We denote by $\mathbb{D}$ the closed unit disk in $\mathbb{C}$, by $\mathbb{N}$ the set of all positive integers and by $\mathcal{B}(X)$ the set of all bounded linear operators on a Banach space X.

Let k be a positive integer and $T_i \in \mathcal{B}(X)$ for all $1 \leq i \leq k$, and let $T = \bigoplus_{i=1}^{k} T_i : \bigoplus_{i=1}^{k} X \to \bigoplus_{i=1}^{k} X$ then we call each operator T_i a component of T.

Definition 2. *An operator T is called k-bitransitive if there exist $T_1, T_2, \cdots T_k \in \mathcal{B}(X)$ such that $T = \bigoplus_{i=1}^{k} T_i$ and for any $2k$-tuples $U_1, \cdots, U_k, V_1, \cdots, V_k \subset X$ of nonempty open sets, there exist some $n \in \mathbb{N}$ and $\alpha_1, \cdots, \alpha_k \in \mathbb{D}\setminus \{0\}$ such that*

$$T^n\Big(\bigoplus_{i=1}^{k} \alpha_i U_i\Big) \cap \Big(\bigoplus_{i=1}^{k} V_i\Big) \neq \varnothing.$$

It is clear from Definition 2 above that 1-bitransitive is identical to disk transitive which in turn identical to diskcyclic.

To simplify Definition 2 above, we provide the following definition.

Definition 3. *Let $r \in \mathbb{N}$ be fixed. For each $1 \leq i \leq r$, let T_i be a bounded linear operator on a Banach space X, and A_i, B_i be nonempty subsets of X. Assume that $T = \bigoplus_{i=1}^{r} T_i$, $A = \bigoplus_{i=1}^{r} A_i$ and $B = \bigoplus_{i=1}^{r} B_i$. The junction set from the set A to the set B under T is defined as $J_T(A, B) = \{(n, \alpha_1, \cdots, \alpha_r) \in \mathbb{N} \times \mathbb{D}^r \setminus \{(0, \cdots, 0)\} : T^n(\bigoplus_{i=1}^{r} \alpha_i A_i) \cap (\bigoplus_{i=1}^{r} B_i) \neq \varnothing\}$.*

In Definition 3 above, we sometimes write $J_T(A,B)$ as $J(A,B)$. The next proposition gives an equivalent definition to k-bitransitivity in terms of junction set.

Proposition 1. *Let $T=\bigoplus_{i=1}^k T_i$. Then T is k-bitransitive if and only if for each $1\le i\le k$ and any nonempty open sets U_i and V_i, there exist $\alpha_i\in\mathbb{D}\setminus\{0\}$ and $n\in\mathbb{N}$ such that*

$$(n,\alpha_i)\in J_{T_i}(U_i,V_i).$$

The proof follows immediately by applying the definition of junction sets to Definition 2.

To answer Question 1, we need the following proposition, which gives a set of sufficient conditions for k-bitransitivity.

Proposition 2 (k-bitransitive criterion)**.** *Let $T=\bigoplus_{i=1}^k T_i$, and let $\{n_r\}_{r\in\mathbb{N}}$ be an increasing sequence of positive integers. Suppose that for each $1\le i\le k$ there exist a sequence $\left\{\lambda_{n_r}^{(i)}\right\}\subset\mathbb{D}\setminus\{0\}$, dense sets $X_i,Y_i\subset X$, and a map $S_i:Y_i\to X$ such that for all $(x_1,\cdots,x_k)\in\bigoplus_{i=1}^k X_i$ and $(y_1,\cdots,y_k)\in\bigoplus_{i=1}^k Y_i$, we have*

(i) $\left\|\bigoplus_{i=1}^k \lambda_{n_r}^{(i)}T_i^{n_r}(x_1,\cdots,x_k)\right\|\to 0$,

(ii) $\left\|\bigoplus_{i=1}^k \frac{1}{\lambda_{n_r}^{(i)}}S_i^{n_r}(y_1,\cdots,y_k)\right\|\to 0$,

(iii) $\bigoplus_{i=1}^k T_i^{n_r}S_i^{n_r}(y_1,\cdots,y_k)\to(y_1,\cdots,y_k)$

as $r\to\infty$. Then T is k-bitransitive.

Proof. Let U_i,V_i be open subsets of X for all $1\le i\le k$, then $\bigoplus_{i=1}^k U_i$ and $\bigoplus_{i=1}^k V_i$ are open in $\bigoplus_{i=1}^k X$. Also $\bigoplus_{i=1}^k X_i$ and $\bigoplus_{i=1}^k Y_i$ are dense in $\bigoplus_{i=1}^k X$. Let

$$(x_1,\cdots,x_k)\in\bigoplus_{i=1}^k U_i\cap\bigoplus_{i=1}^k X_i$$

and

$$(y_1,\cdots,y_k)\in\bigoplus_{i=1}^k V_i\cap\bigoplus_{i=1}^k Y_i.$$

Suppose that $z_r=(x_1,\cdots,x_k)+\bigoplus_{i=1}^k\frac{1}{\lambda_{n_r}^{(i)}}S_i^{n_r}(y_1,\cdots,y_k)$. By (ii), as $r\to\infty$ we have

$$\|z_r-(x_1,\cdots,x_k)\|=\left\|\bigoplus_{i=1}^k\frac{1}{\lambda_{n_r}^{(i)}}S_i^{n_r}(y_1,\cdots,y_k)\right\|\to 0. \tag{1}$$

Since

$$\bigoplus_{i=1}^k\lambda_{n_r}^{(i)}T_i^{n_r}(z_r)=\bigoplus_{i=1}^k\lambda_{n_r}^{(i)}T_i^{n_r}\left((x_1,\cdots,x_k)+\bigoplus_{i=1}^k\frac{1}{\lambda_{n_r}^{(i)}}S_i^{n_r}(y_1,\cdots,y_k)\right),$$

then by (i) and (iii), we have

$$\left\|\bigoplus_{i=1}^k\lambda_{n_r}^{(i)}T_i^{n_r}(z_r)-(y_1,\cdots,y_k)\right\|=\left\|\bigoplus_{i=1}^k\lambda_{n_r}^{(i)}T_i^{n_r}(x_1,\cdots,x_k)\right\|\to 0, \tag{2}$$

as $r \to \infty$. From Equations (1) and (2), there exists $N \in \mathbb{N}$ such that $z_N \in \bigoplus_{i=1}^{k} U_i$ and $\bigoplus_{i=1}^{k} \lambda_{n_r}^{(i)} T_i^{n_r}(z_N) \in \bigoplus_{i=1}^{k} V_i$, that is,

$$\bigoplus_{i=1}^{k} \lambda_{n_r}^{(i)} T_i^{n_r} \left(\bigoplus_{i=1}^{k} U_i \right) \cap \bigoplus_{i=1}^{k} V_i \neq \varnothing \text{ for all } r \geq N,$$

which is equivalent to

$$(T_1 \oplus \cdots \oplus T_k)^{n_r} (\lambda_{n_r}^{(1)} U_1 \oplus \cdots \oplus \lambda_{n_r}^{(k)} U_k) \cap (V_1 \oplus \cdots \oplus V_k) \neq \varnothing \quad \text{for all} \quad r \geq N.$$

That is,

$$(n_r, \lambda_{n_r}^{(i)}) \in J_{T_i}(U_i, V_i) \quad \text{for all} \quad 1 \leq i \leq k.$$

By Proposition 1, T is k-bitransitive. □

The following theorem gives a partial answer to Question 1.

Theorem 1. *If k operators satisfy diskcyclic criterion for the same increasing sequence of positive integers $\{n_r\}_{r \in \mathbb{N}}$, then their direct sum is a k-bitransitive operator.*

Proof. Let $T_i \in \mathcal{B}(X)$ satisfies diskcyclic criterion with respect to the same increasing sequence of positive integers $\{n_r\}_{r \in \mathbb{N}}$ for all $1 \leq i \leq k$ [2, Theorem 2.6]. Then for each $1 \leq i \leq k$, there exists a sequence $\left\{ \lambda_{n_r}^{(i)} \right\}_{r \in \mathbb{N}} \in \mathbb{D} \setminus \{0\}$, two dense sets D_i, D_i' and a map S_i such that for all $x_i \in D_i$ and $y_i \in D_i'$, we have

$$\left\| \lambda_{n_r}^{(i)} T_i^{n_r} x_i \right\| \to 0, \tag{3}$$

$$\left\| \frac{1}{\lambda_{n_r}^{(i)}} S_i^{n_r} y_i \right\| \to 0, \tag{4}$$

$$T_i^{n_r} S_i^{n_r} y_i \to y_i \tag{5}$$

as $r \to \infty$. By Equation (3), we get $\sum_{i=1}^{k} \left\| \lambda_{n_r}^{(i)} T_i^{n_r} x_i \right\| \to 0$; that is,

$$\left\| \bigoplus_{i=1}^{k} \lambda_{n_r}^{(i)} T_i^{n_r} (x_1, \cdots, x_k) \right\| \to 0 \tag{6}$$

as $r \to \infty$. Also by Equation (4), we get $\sum_{i=1}^{k} \left\| \frac{1}{\lambda_{n_r}^{(i)}} S_i^{n_r} y_i \right\| \to 0$; that is,

$$\left\| \bigoplus_{i=1}^{k} \frac{1}{\lambda_{n_r}^{(i)}} S_i^{n_r} (y_1, \cdots, y_k) \right\| \to 0 \tag{7}$$

as $r \to \infty$. Finally, by Equation (5), we get $(T_1^{n_r} S_1^{n_r} y_1, \cdots, T_k^{n_r} S_k^{n_r} y_k) \to (y_1, \cdots, y_k)$; that is,

$$\bigoplus_{i=1}^{k} T_i^{n_r} S_i^{n_r} (y_1, \cdots, y_k) \to (y_1, \cdots, y_k) \tag{8}$$

as $r \to \infty$. By Proposition 2, we get $T = \bigoplus_{i=1}^{k} T_i$ is k-bitransitive. □

To give another partial answer to Question 1, we define another class of operators which is called compound operators.

Definition 4. *Let $T \in \mathcal{B}(X)$. Then T is called compound if for any nonempty open sets U and V, there exist some $N \in \mathbb{N}$ and a sequence $\{\alpha_n\}_{n\in\mathbb{N}} \in \mathbb{D}\setminus\{0\}$ such that*

$$T^n(\alpha_n U) \cap V \neq \varnothing$$

for all $n \geq N$.

The following theorem gives another partial answer to Question 1. First, we need the following lemma.

Lemma 1. *If $T \in \mathcal{B}(X)$ is diskcyclic, then there exist an increasing sequence of positive integers $\{m_j\}_{j\in\mathbb{N}}$ and a sequence $\left\{\gamma_{m_j}\right\} \subset \mathbb{D}\setminus\{0\}$ such that $\left\{(m_j, \gamma_{m_j}) : j \in \mathbb{N}\right\} \subseteq J(U,V)$ for any two nonempty open sets $U, V \subset X$.*

Proof. Let $(n_1, \alpha_1) \in J(U,V)$, and let $W = U \cap T^{-n_1}\frac{1}{\alpha_1}V$. Since W is open set, then there exist $n_2 \in \mathbb{N}$ and $\alpha_2 \in \mathbb{D}$ such that $(n_2, \alpha_2) \in J(W,W)$, that is,

$$T^{n_2}\alpha_2 U \cap T^{n_2-n_1}\frac{\alpha_2}{\alpha_1}V \cap U \cap T^{-n_1}\frac{1}{\alpha_1}V \neq \varnothing.$$

It follows that

$$T^{n_2}\alpha_1\alpha_2 U \cap T^{-n_1}V \neq \varnothing.$$

Now, we have

$$T^{n_1+n_2}\alpha_1\alpha_2 U \cap V = T^{n_1}(T^{n_2}\alpha_1\alpha_2 U \cap T^{-n_1}V) \neq \varnothing,$$

that is,

$$(n_1+n_2, \alpha_1\alpha_2) \in J(U,V).$$

By continuing the same process, we get $(\sum_{i=1}^{j} n_i, \prod_{i=1}^{j}\alpha_i) \in J(U,V)$ for any $j, n_i \in \mathbb{N}$ and $\alpha_i \in \mathbb{D}$. Let $m_j = \sum_{i=1}^{j} n_i$ and $\gamma_{m_j} = \prod_{i=1}^{j}\alpha_i$ for all $j \in \mathbb{N}$, then

$$\left\{(m_j, \gamma_{m_j}) : j \in \mathbb{N}\right\} \subseteq J(U,V).$$

□

Theorem 2. *Let $T = \bigoplus_{i=1}^{k} T_i$. If every component of T is disk transitive and at least $(k-1)$ of them are compound, then T is k-bitransitive.*

Proof. Without loss of generality, we suppose that $k = 2$ and T_1 is compound. Let U_1, U_2, V_1, V_2 be nonempty open sets, by hypothesis there exist $N_1, N_2 \in \mathbb{N}$, $\alpha_1 \in \mathbb{D}\setminus\{0\}$ and a sequence $\{\beta_n : n \in \mathbb{N}\} \subset \mathbb{D}\setminus\{0\}$ such that

$$T_2^{N_1}\alpha_1 U_1 \cap U_2 \neq \varnothing \text{ and } T_1^n \beta_n V_1 \cap V_2 \neq \varnothing$$

for all $n \geq N_2$. By Lemma 1, there exist $N \in \mathbb{N}$ and $\alpha \in \mathbb{D}\setminus\{0\}$ such that

$$T_2^N \alpha U_1 \cap U_2 \neq \varnothing \text{ and } T_1^N \beta_N V_1 \cap V_2 \neq \varnothing.$$

It follows that

$$(T_1 \oplus T_2)^N(\alpha U_1 \oplus \beta_N V_1) \cap (U_2 \oplus V_2) \neq \varnothing.$$

Hence T is 2-bitransitive. □

It is clear that every compound operator is diskcyclic. A special case of compound operator is when $\alpha_n = 1$ for all $n \geq N$, and it is called mixing operators (see [6]). Therefore every mixing operator is compound. However, not every compound operator is mixing as shown in the following example. First, we need the following proposition which give sufficient conditions for an operator to be compound.

Proposition 3. *Let $T \in \mathcal{B}(X)$, suppose that there exist a sequence $\{\lambda_n\}_{n\in\mathbb{N}} \subset \mathbb{D}\setminus\{0\}$, two dense sets D_1 and D_2 in X, and a sequence of maps $S_n : D_2 \to X$ and such that*

(i) $\|\lambda_n T^n x\| \to 0$ *for any* $x \in D_1$,

(ii) $\left\|\frac{1}{\lambda_n} S_n y\right\| \to 0$ *for any* $y \in D_2$,

(iii) $T^n S_n y \to y$ *for any* $y \in D_2$

as $n \to \infty$. Then T is compound and it is called compound with respect to the sequence $\{\lambda_n\}$.

Proof. Suppose that U and V be two nonempty open sets. Let $x \in U \cap D_1$ and $y \in V \cap D_2$. Let N be a large positive integer such that $z = x + \frac{1}{\lambda_N} S_N y$, then by hypothesis we get

$$\|z - x\| = \left\|\frac{1}{\lambda_N} S_N y\right\| \to 0 \quad \text{and} \quad \left\|\lambda_N T^N z - y\right\| = \left\|\lambda_N T^N x\right\| \to 0.$$

Thus $T^n \lambda_n U \cap V \neq \varnothing$ for all $n \geq N$. So, T is compound. □

The following proposition gives another criterion for compound operators without the need of the scalar sequence.

Proposition 4. *Let $T \in \mathcal{B}(X)$. If there exist two dense sets D_1 and D_2 in X, and a sequence of maps $S_n : D_2 \to X$ such that*

(i) $\|T^n x\| \, \|S_n y\| \to 0$ *for all* $x \in D_1$ *and* $y \in D_2$,

(ii) $\|S_n y\| \to 0$ *for all* $y \in D_2$,

(iii) $T^n S_n y \to y$ *for all* $y \in D_2$

as $n \to \infty$. Then T is compound.

The proof of Proposition 4 is followed by showing that both compound criteria in Propositions 3 and 4 are equivalent by using the same lines in [2, Proposition 2.8].

Example 1. *Let T be a bilateral forward weighted shift on $\ell_p, 1 \leq p < \infty$, with the weight sequence*

$$w_n = \begin{cases} R_1, & \text{if } n \geq 0, \\ R_2, & \text{if } n < 0, \end{cases}$$

where $R_1, R_2 \in \mathbb{R}^+; 1 < R_1 < R_2$. Then T is compound not mixing.

Proof. By applying [3, Corollary 2.15] and taking $\{n_r\}_{r\in\mathbb{N}} = \{n\}_{n\in\mathbb{N}}$, we get

$$\lim_{n\to\infty}\prod_{k=1}^{n}\frac{1}{w_{-k}} = \lim_{n\to\infty}\prod_{k=1}^{n}\frac{1}{R_2} = \lim_{n\to\infty}\frac{1}{R_2^n} = 0$$

and

$$\lim_{n\to\infty}\left(\prod_{k=1}^{n} w_k\right)\left(\prod_{k=1}^{n}\frac{1}{w_{-k}}\right) = \lim_{n\to\infty}\left(\prod_{k=1}^{n} R_1\right)\left(\prod_{k=1}^{n}\frac{1}{R_2}\right) = \lim_{n\to\infty}(R_1^n)(\frac{1}{R_2^n}) = 0.$$

It follows that T satisfies diskcyclic criterion with respect to the sequence $\{n\}_{n\in\mathbb{N}}$. Then, by Proposition 4, T is compound. Now, since

$$\lim_{n\to\infty}\left(\prod_{k=1}^{n} w_k\right) = \infty,$$

then by [8, Theorem 3.2] T is not topological transitive and so not mixing. □

The following theorem extends the Godefroy-Shapiro Criterion [1, Theorem 1.3] for mixing operators to compound operators.

Theorem 3. *Let $T \in \mathcal{B}(X)$. If there exists a positive integer $p \geq 1$ such that*

$$A = \text{span}\,\{x \in X : Tx = \alpha x \text{ for some } \alpha \in \mathbb{C}; |\alpha| < p\}$$

and

$$B = \text{span}\,\{y \in X : Ty = \lambda y \text{ for some } \lambda \in \mathbb{C}; |\lambda| > p\}$$

are dense in X, then T is compound.

Proof. Let U and V be nonempty open sets in X. Since A and B are dense, then there exist $x \in A \cap U$ and $y \in B \cap V$. Then $x = \sum_{i=1}^{k} a_i x_i$ and $y = \sum_{i=1}^{k} b_i y_i$, where $a_i, b_i \in \mathbb{C}$ for all $1 \leq i \leq k$. Also, $Tx_i = \alpha_i x_i$ and $Ty_i = \lambda_i y_i$, where $|\alpha_i| < p$ and $|\lambda_i| > p$ for all $1 \leq i \leq k$. Let $c \in \mathbb{C}$ be a scalar such that $p \leq |c| < |\lambda_i|$ for all $1 \leq i \leq k$, and let

$$z_n = \sum_{i=1}^{k} b_i(\frac{c}{\lambda_i})^n y_i \quad \text{for all} \quad n \geq 0.$$

Then

$$\frac{1}{c^n}T^n x = \sum_{i=1}^{k} a_i(\frac{\alpha_i}{c})^n x_i \to 0 \quad \text{and} \quad z_n \to 0 \quad \text{as} \quad n \to \infty.$$

Also, $\frac{1}{c^n}T^n z_n = y$ for all $n \geq 0$. It follows that there is a positive integer k such that for all $n \geq k$, we have

$$x + z_n \in U \quad \text{and} \quad \frac{1}{c^n}T^n(x+z_n) = \frac{1}{c^n}T^n x + \frac{1}{c^n}T^n z_n \in V \quad \text{for all} \quad n \geq k.$$

Therefore, $\frac{1}{c^n}T^n U \cap V \neq \varnothing$ for all $n \geq k$. It follows that T is compound. □

Note that in the above theorem, if $p = 1$, then it will be a Godefroy-Shapiro criterion for mixing operators [1, Theorem 1.3].

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В цій статті ми вводимо нові класи операторів у комплексних банахових просторах, які ми називаємо k-бітранзитивними операторами і операторами сполучення для вивчення прямих сум дискциклічних операторів. Запропоновано набір достатніх умов для того, щоб оператор був k-бітранзитивним чи оператором сполучення. Також встановлено зв'язок між операторами топологічного змішування і операторами сполучення. Також розширено критерій Ґодефруа-Шапіро для операторів топологічного змішування на випадок операторів сполучення.

Ключові слова і фрази: гіперциклічні оператори, дискциклічні оператори, оператори слабкого змішування, прямі суми.