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Reconstruction of Computed Tomography images using an optimal quadrature formula

Babaev S. S.[1,2], Mahkamova D. T.[3], Ulikov Sh. Sh. [4]
[1]Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan;
[2]Bukhara state university, Bukhara, Uzbekistan; b.samandar@mail.ru
[3]National University of Uzbekistan, Tashkent, Uzbekistan; d.mahkamova@list.ru
[3]Fergana state university, Fergana, Uzbekistan; sh.ulikov@mail.ru

In this paper we construct the optimal quadrature formula in the sense of Sard for numerical integration of the integral $\int\limits_a^b e^{2\pi i\omega x}\varphi(x)dx$ with $\omega \in R$ in the space $W_2^{(2,1)}[a,b]$ of complex-valued functions which are square integrable with m-th order derivative. We obtain the explicit formulas for optimal coefficients using the discrete analogue of the differential operator $d^4/dx^4 - d^2/dx^2$. The order of convergence of the optimal quadrature formula is $O(h^2)$. We apply the optimal quadrature formula for reconstruction of Computed Tomography images by approximating Fourier transforms in the filtered back-projection formula.

We have done some numerical experiments on phantoms and have compared them with the results of a built-in function of MATLAB 2019a, iradon. Numerical results show that the quality of the reconstructed images with the constructed optimal quadrature formula is better than that of the results obtained by iradon.

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On Mersenne-Fibonacci relations

Frontczak R.[1], Goy T.[2]
[1]Landesbank Baden-Württemberg, Stuttgart, Germany; robert.frontczak@lbbw.de
[2]Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine; taras.goy@pnu.edu.ua

A *Mersenne number*, denote by M_n, is a number of the form $M_n = 2^n - 1$, $n \geq 0$. The Mersenne sequence $\{M_n\}_{n\geq 0}$ can be defined recursively as follows $M_0 = 1$, $M_1 = 1$, and $M_n = 3M_{n-1} - 2M_{n-2}$ for $n \geq 2$.

Mersenne numbers are popular research objects because of their interesting properties. For instance, Mersenne numbers are numbers with the following representation in the binary system: $(1)_2$, $(11)_2$, $(111)_2$, $(1111)_2$, ... Also, the Mersenne number sequence contains primes, the so called *Mersenne primes* of the form $2^n - 1$. A simple calculation shows that if M_n is a prime number, then n is a prime number, though not all M_n are prime. Mersenne primes are also connected to perfect numbers.

More information about Mersenne numbers and there properties can be taken from the papers [1, 4-6] and references contained therein.

The main goal of the present note is to reveal connection between Mersenne numbers and Fibonacci numbers, defined by the recurrence: for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$, $F_1 = 1$.

Our first result provides a relation between Mersenne and Fibonacci numbers. The method of proof is similar to that in [2, 3].

Theorem 1. *For $n \geq 1$, the following formulas hold*

$$F_n = M_n - \sum_{k=1}^{n-1} (2F_{n-k} - 3F_{n-k-1}) M_k,$$

$$F_{2n+1} + 2F_{2n-1} = M_{2n-1} + 3 - \sum_{k=1}^{n-1} \left(2F_{2(n-k)} - F_{2(n-k)-1}\right) M_{2k-1},$$

$$3F_{2n} = M_{2n} - \sum_{k=1}^{n-1} \left(2F_{2(n-k)} - 3F_{2(n-k-1)}\right) M_{2k}.$$

Next, we derive some connection formulas between Mersenne and Fibonacci numbers involving binomial coefficients.

Theorem 2. *For $n \geq 1$, the following formulas hold*

$$\sqrt{5}F_n = \left(\frac{1-3\sqrt{5}}{2}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(-\frac{15+\sqrt{5}}{22}\right)^k M_k,$$

$$M_n = \sqrt{5}\left(\frac{15-\sqrt{5}}{10}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(\frac{1+3\sqrt{5}}{22}\right)^k F_k. \tag{3}$$

Note, formula (1) may be rewritten in terms of the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$ as follows

$$M_n = \left(\frac{\varphi^2+2}{(2\varphi-1)\varphi}\right)^n \sum_{k=1}^{n} \binom{n}{k} \frac{\varphi^{2k} - (-1)^k}{(\varphi^2+2)^k}.$$

Theorem 3. *For $n \geq 1$, the following formulas hold*

$$\sqrt{5}\sum_{k=1}^{n} \binom{n}{k} F_k = \left(\frac{3-3\sqrt{5}}{2}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(-\frac{5+\sqrt{5}}{6}\right)^k M_k,$$

$$\sqrt{5}\sum_{k=1}^{n} \binom{n}{k} (-2)^k F_k = (3\sqrt{5})^n \sum_{k=1}^{n} \binom{n}{k} \left(-\frac{2}{3}\right)^k M_k.$$

In a similar manner, we obtain some relations between even (odd) indexed Mersenne and Fibonacci numbers.

Theorem 4. *For $n \geq 1$, we have*

$$\sqrt{5}F_n = \left(\frac{3-5\sqrt{5}}{6}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(-\frac{25+3\sqrt{5}}{58}\right)^k M_{2k},$$

$$M_{2n} = \sqrt{5}\left(\frac{25-3\sqrt{5}}{10}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(\frac{9+15\sqrt{5}}{58}\right)^k F_k.$$

Theorem 5. *For $n \geq 1$, it holds that*

$$\sqrt{5}F_n = \left(\frac{3-5\sqrt{5}}{6}\right)^n \sum_{k=0}^{n} \binom{n}{k} \left(-\frac{25+3\sqrt{5}}{58}\right)^k M_{2k+1} - \left(\frac{1+\sqrt{5}}{2}\right)^n,$$

$$M_{2n+1} = \sqrt{5}\left(\frac{25-3\sqrt{5}}{10}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(\frac{9+15\sqrt{5}}{58}\right)^k F_k + 4^n.$$

Theorem 6. *For $n \geq 1$, we have*

$$\sqrt{5}F_{2n} = \left(\frac{9-5\sqrt{5}}{6}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(-\frac{25+9\sqrt{5}}{22}\right)^k M_{2k},$$

$$M_{2n} = \sqrt{5}\left(\frac{25-9\sqrt{5}}{10}\right)^n \sum_{k=1}^{n} \binom{n}{k} \left(\frac{27+15\sqrt{5}}{22}\right)^k F_{2k}.$$

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