

If the boundary  $\partial B$  is smooth enough, we are able to estimate the rate of convergence in (1). For example, if  $\partial B$  is the union of  $m$  algebraic surfaces of degree at most  $d$ , then for  $n \geq 3$  the rate of convergence in (1) can be estimated as  $C_{n,m,d} Q^{-1}$ , where  $C_{n,m,d}$  depends only on  $n, m, d$  and on the height function  $h$ .

It turns out that the function  $\rho_{k,l}$  coincides with the correlation function of the roots of some specific random polynomial. Moreover, there exist explicit symmetric functions  $\psi_m : \mathbb{C}^m \rightarrow \mathbb{R}_+$  (depending on  $n$ ) such that all the functions  $\rho_{k,l}$  satisfy the equality

$$\rho_{k,l}(x_1, \dots, x_k, z_1, \dots, z_l) = 2^l \psi_{k+2l}(x_1, \dots, x_k, z_1, \dots, z_l, \bar{z}_1, \dots, \bar{z}_l).$$

Every  $\psi_m$  can be represented as the product of  $\prod_{1 \leq i < j \leq m} |z_i - z_j|$  and a continuous function.

- [1] F. GÖTZE, D.V. KOLEDA, D.N. ZAPOROZHETS. Joint distribution of conjugate algebraic numbers: a random polynomial approach // arXiv:1703.02289, 2017.

## ON PELL IDENTITIES WITH MULTINOMIAL COEFFICIENTS

**T. Goy** (Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine)

The *Pell numbers* are an integer sequence defined recursively by  $P_0 = 0$ ,  $P_1 = 1$ , and  $P_n = 2P_{n-1} + P_{n-2}$  for all  $n \geq 2$  (see [1] and the references given there).

**Proposition.** *Let  $n \geq 1$ , except when otherwise. The following formulas hold:*

$$\begin{aligned} \sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_1^{t_1} P_2^{t_2} \cdots P_n^{t_n} &= -F_n, \\ \sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_1^{t_1} P_3^{t_2} \cdots P_{2n-1}^{t_n} &= -4 \cdot 5^{n-2}, \quad n \geq 2, \end{aligned}$$

$$\begin{aligned}
\sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_2^{t_1} P_4^{t_2} \dots P_{2n}^{t_n} &= \frac{(2 - \sqrt{3})^n - (2 + \sqrt{3})^n}{\sqrt{3}}, \\
\sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_3^{t_1} P_5^{t_2} \dots P_{2n+1}^{t_n} &= -4, \quad n \geq 2, \\
\sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_2^{t_1} P_3^{t_2} \dots P_{n+1}^{t_n} &= 0, \quad n \geq 3, \\
\sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_0^{t_1} P_1^{t_2} \dots P_{n-1}^{t_n} &= -2^{n-2}, \quad n \geq 2, \\
\sum_{(t_1, \dots, t_n)} (-1)^T p_n(t) P_3^{t_1} P_4^{t_2} \dots P_{n+2}^{t_n} &= (-1)^n F_{2n+3}, \\
\sum_{(t_1, \dots, t_n)} p_n(t) P_0^{t_1} P_1^{t_2} \dots P_{n-1}^{t_n} &= \frac{(1 + \sqrt{3})^{n-1} - (1 - \sqrt{3})^{n-1}}{2\sqrt{3}}, \\
\sum_{(t_1, \dots, t_n)} p_n(t) P_0^{t_1} P_2^{t_2} \dots P_{2n-2}^{t_n} &= \frac{(3 + \sqrt{10})^{n-1} - (3 - \sqrt{10})^{n-1}}{\sqrt{10}},
\end{aligned}$$

where the summation is over integers  $t_i \geq 0$  satisfying  $t_1 + 2t_2 + \dots + nt_n = n$ ,  $T = t_1 + \dots + t_n$ ,  $F_n$  is the  $n$ -th Fibonacci number, and  $p_n(t) = \frac{(t_1 + \dots + t_n)!}{t_1! \dots t_n!}$  is the multinomial coefficient.

[1] T. KOSHY. Pell and Pell-Lucas Numbers with Applications. Springer, 2014.

## **DISTRIBUTION OF VALUES JORDAN'S FUNCTION IN RESIDUE CLASSES**

**L. A. Gromakovskaja, B. M. Shirokov**

(Petrozavodsk State University, Petrozavodsk, Russia)

The pair of integers  $a, b$  is called primitive modulo integer  $n$  if the greatest common divisor  $(a, b, n) = 1$ . Let's denote by  $J(n)$  the number of incongruent primitive pairs of integers modulo  $n$ . This function is called Jordan's function. The properties of  $J(n)$  were studied in [1].

Let's denote by  $S(x, r, f)$  for  $x > 0$ , integers  $r, N$ ,  $(r, N) = 1$ , and integral-value arithmetical function  $f(n)$  the number of integers  $n > 0$  for which  $f(n) \equiv r$