Taras Goy (Vasyl Stefanyk Precarpathian National University)

Connection formulas between the Oresme and Fibonacci numbers

In [Horadam A.F., Oresme numbers, *Fibonacci Quart.* **3**, 267–271 (1974)], Horadam presented a history of the sequence

$$\{O_n\}_{n\geq 1} = \left\{\frac{n}{2^n}\right\}_{n\geq 1}$$

attributed to French philosopher and naturalist Nicole Oresme.

We consider some families of Toeplitz-Hessenberg determinants the entries of which are Oresme numbers. These determinant formulas may also be rewritten as identities involving sums of products of Oresme numbers and multinomial coefficients.

Our approach is similar in spirit to [Goy T., On identities with multinomial coefficients for Fibonacci-Narayana sequence, *Ann. Math. Inform.* **49**, 75–84 (2018); Goy T., On new identities for Mersenne numbers, *Appl. Math. E-Notes* **18**, 100–105 (2018)].

Recall that Fibonacci sequence $\{F_n\}_{n\geq 0}$ is defined by the recurrence $F_n = F_{n-1} + F_{n-2}$, where $F_0 = 0$, $F_1 = 1$ and $n \geq 2$.

Theorem: *For all $n \geq 1$, the following formulas hold:*

$$F_{2n} = 2^n \cdot \sum_{(s_1,\ldots,s_n)} p_n(s) O_1^{s_1} O_2^{s_2} \cdots O_n^{s_n},$$

$$F_{3n-1} = 2^{2n-1} \cdot \sum_{(s_1,\ldots,s_n)} p_n(s) O_1^{s_1} O_3^{s_2} \cdots O_{2n-1}^{s_n},$$

where the summation is over nonnegative integers s_i satisfying $s_1 + 2s_2 + \cdots + ns_n = n$ and $p_n(s) = \binom{s_1+\cdots+s_n}{s_1,\ldots,s_n}$.

Buket Eren Gökmen (Galatasaray University)

On the Markov Equation and the outer automorphism of $PGL(2,\mathbb{Z})$

The Markov numbers are the unions of the solutions $(x;y;z) \in \mathbb{Z}_+^3$ to the Markov equation $x^2+y^2+z^2 = 3xyz$. Markov (1879) showed that all possible solutions are generated from $(1;1;1)$ by a simple algorithm. These numbers arise in many different contexts such as binary quadratic forms, hyperbolic geometry, combinatorics etc. with beautiful interconnections. Our interest is in the set of continued fraction expansions of Markov quadratic irrationals arised from Markov numbers. However, there is a fundamental involution of the real line called Jimm [Uludağ, M. and Ayral, H. (2016) On the Involution of the Real Line Induced by Dyers Outer Automorphism of $PGL(2,\mathbb{Z})$ arXiv :1605.03717 [math.AG]] induced by the outer automorphism of the extended modular group $PGL(2;\mathbb{Z})$. Its action on the real line, explicitly on continued fraction expansion, inspired us that this involution must play a role in Markov's theory. The main goal of this talk is to show the possible relation between Jimm involution and the Markov theory. More precisely, we will first present a method to

nd directly the quadratic form of the image of Markov irrationals under Jimm involution by using the fact coming from geometry of Markov numbers. Then we will show a general form of the image of the subset of Markov quadratic irrationals in relation with Fibonacci numbers [Eren Gokmen B. and Uludağ, M. (2018) The Conumerator arXiv:1811.01881v1 [math.NT]].

*(joint with **A. Muhammed Uludağ**, Galatasaray University)*

Haydar Göral (Department of Mathematics, Faculty of Science, Dokuz Eylül University)

Fractions in Number Fields

Any finite sum of distinct unit fractions is called an Egyptian fraction. A restricted Egyptian fraction means that the sum consists of at most k unit fractions for some fixed positive integer k, and we also allow repetitions. In this talk, we consider the unit fraction representations in the ring of integers of a given number field. In particular, we prove that finitely many representations of 1 as a sum of unit fractions (as restricted Egyptian fractions) determines the field of rational numbers among all real number fields.

*(joint with **Doğa Can Sertbaş**, Department of Mathematics, Faculty of Sciences, Cumhuriyet University)*
