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ON NEW FAMILIES OF THE TETRANACCI IDENTITIES

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The tetranacci numbers are a more general version of Fibonacci numbers and start with four predetermined terms, each term afterwards being the sum of the preceding four terms.

The tetranacci sequence $\{t_n\}_{n\geq 0}$ is the sequence of integersdefined by the initial values $t_0 = t_1 = t_2 = 0$, $t_3 = 1$ and the recurrence

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} + t_{n-4}, \qquad n \geq 4.$$

For Toeplitz-Hessenberg determinants entries of which are tetranacci numbers we use Trudi's formula (see [1–4] for more details) to obtain new identities involving sums of products of tetranacci numbers and multinomial coefficients.

Let F_n and T_n be the Fibonacci and tribonacci numbers, defined by recurrences

$$F_0 = 0, \quad F_1 = 1, F_n = F_{n-1} + F_{n-2}, \quad n \geq 2,$$

$$T_0 = T_1 = 0, \quad T_2 = 1, \qquad T_n = T_{n-1} + T_{n-2} + T_{n-3}, \quad n \geq 3,$$

respectively.

Let us denote $\sigma_n = s_1 + 2s_2 + \ldots + ns_n$, $|s| = s_1 + s_2 + \ldots + s_n$, and $p_n(s) = \dfrac{(s_1 + s_2 + \ldots + s_n)!}{s_1! s_2! \ldots s_n!}$.

Theorem. *The following formulas hold*

$$\sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) t_0^{s_1} t_1^{s_2} \ldots t_{n-1}^{s_n} = T_{n-2}, \qquad n \geq 2,$$

$$\sum_{\sigma_n=n}(-1)^{|s|}p_n(s)t_2^{s_1}t_3^{s_2}\dots t_{n+1}^{s_n}=\sum_{i=1}^{\lfloor n/2\rfloor}(-1)^i F_{n-2i+1}, \qquad n\ge 1,$$

$$\sum_{\sigma_n=n}(-1)^{|s|+1}p_n(s)t_5^{s_1}t_7^{s_2}\dots t_{2n+3}^{s_n}=F_{n+2}, \qquad n\ge 3,$$

$$\sum_{\sigma_n=n}(-1)^{|s|}p_n(s)t_4^{s_1}t_5^{s_2}\dots t_{n+3}^{s_n}=0, \qquad n\ge 5,$$

$$\sum_{\sigma_n=n}p_n(s)t_5^{s_1}t_6^{s_2}\dots t_{n+4}^{s_n}=\frac{(-1)^n+(-1)^{\lfloor 3n/2\rfloor}}{2}, \qquad n\ge 2,$$

$$\sum_{\sigma_n=n}p_n(s)t_0^{s_1}t_1^{s_2}\dots t_{n-1}^{s_n}=\frac{2+(-1)^n}{(-1)^{\lfloor n/2\rfloor}10}+\frac{5(-1)^n+2^n}{30}, \qquad n\ge 1,$$

$$\sum_{\sigma_n=n}p_n(s)t_1^{s_1}t_2^{s_2}\dots t_n^{s_n}=\sum_{i=0}^{\lfloor (n-3)/2\rfloor}\sum_{j=0}^{n-2i-3}\binom{n-3-i-j}{i}\binom{2i}{j}, \qquad n\ge 1,$$

$$\sum_{\sigma_n=n}(-1)^{|s|+1}p_n(s)t_3^{s_1}t_4^{s_2}\dots t_{n+2}^{s_n}$$

$$=\sum_{i=0}^{\lfloor (n-1)/2\rfloor}\sum_{j=0}^{\lfloor (n-1)/2\rfloor}\binom{j}{2i+3j-n+1}\binom{2i+3j-n+1}{i}, \qquad n\ge 1,$$

where the summation is over nonnegative integers s_i *satisfying Diophantine equation* $s_1+2s_2+\dots+ns_n=n$.

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