# AUTOMORPHISM GROUPS OF SUPEREXTENSIONS OF SEMIGROUPS

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The through study of various extensions of semigroups was started in [12] and continued in [1]-[10], [13]-[19]. The largest among these extensions is the semigroup $\upsilon(S)$ of all upfamilies on a semigroup $S$. A family $\mathcal{M}$ of non-empty subsets of a set $X$ is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of $X$ belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of non-empty subsets of $X$ generates the upfamily $\{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B}\rangle$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set $X$ is called the *Stone-Čech compactification* of $X$, see [20]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [12] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a \in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains as subsemigroups many other important extensions of $S$. In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies, see [11, 12]. An upfamily $\mathcal{L}$ of subsets of $S$ is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of $S$ is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on $S$ that contains $\mathcal{M}$. It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of $S$, see [21, 22].

Given a semigroup $S$ we shall discuss the algebraic structure of the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of $S$. We show that any automorphism of a semigroup $S$ can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup $S$ contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$. We describe automorphism groups of superextensions of groups, finite monogenic semigroups, null semigroups, almost null semigroups, right zero semigroups, left zero semigroups and all three-element semigroups.

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