Использование пакета MATLAB в учебном процессе позволит совершенствовать формы и методы самостоятельной работы студента при изучении курса высшей математики. Появляется возможность больше внимания уделить рассмотрению узловых вопросов курса за счет уменьшения затрат на рутинную вычислительную работу, развиваются творческие способности студентов.

**Литература**


3. Ануфриев И.Е. Самоучитель MatLab 5.3/6.х. –Спб.: БХВ Санкт-Петербург, 2002,-736 с.
4. Вентцель Е.С. Теория вероятностей. –М.: Высш. шк., 2001.-575 с.
5. Потемкин В.Г. Система MatLab 5 для студентов. Справочное пособие. –М.: ДИАЛОГ-МИФИ, 1998. -314 с.


**УДК 511.176**

## Some combinatorial identities for two-periodic Fibonacci sequence


**T. Goy**

**Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine**

**tarasgoy@yahoo.com**


Consider the Fibonacci sequence $\{F_n\}_{n\geq 0}$ having initial conditions $F_0 = 0$, $F_1 = 1$ and recurrence relation $F_n = F_{n-1} + F_{n-2}$ with $n \geq 2$. The Fibonacci sequence has been generalized in many ways, some by preserving the initial conditions, and others by preserving the recurrence relation [3, 5]. We study a generalization $\{q_n\}_{n\geq 0}$ with initial conditions $q_0 = 0$ and $q_1 = 1$ which is defined by the recurrence

$$q_n = \begin{cases} aq_{n-1} + q_{n-2}, & \text{if } n \text{ is even,} \\ bq_{n-1} + q_{n-2}, & \text{if } n \text{ is odd,} \end{cases}$$

where $a$ and $b$ are nonzero real numbers. These sequences arise in a natural way in the study of continued fractions of quadratic irrationals and combinatorics on words or dynamical system theory (see, for example, [1, 2, 4] for more details).

Some well-known sequences are special cases of this generalization. Classical Fibonacci sequence is a special case of $\{q_n\}_{n\geq 0}$ with $a = b = 1$. The Pell sequence is

$\{q_n\}_{n\geq 0}$ with $a = b = 2$ and the $k$-Fibonacci sequence is $\{q_n\}_{n\geq 0}$ with $a = b = k$, for some positive integer $k$.

Explicit expressions for the first ten terms of sequence $\{q_n\}_{n\geq 0}$ are

$q_0 = 0$, $q_1 = 1$, $q_2 = a$, $q_3 = ab+1$, $q_4 = a^2b+2a$, $q_5 = a^2b^2+3ab+1$, $q_6 = a^3b^2+4a^2b+3a$, $q_7 = a^3b^3+5a^2b^2+6ab+1$, $q_8 = a^4b^3+6a^3b^2+10a^2b+4a$, $q_9 = a^4b^4+7a^3b^3+15a^2b^2+10ab+1$.

Using Toeplitz-Hessenberg determinants with special entries $q_i$, we derive the following formulas with multinomial coefficients.

**Proposition 1.** *Let* $n \geq 2$*, except when noted otherwise. Then*

$$\sum_{s_1+2s_2+\cdots+ns_n=n} a^{-s_1-\cdots-s_n} p_n(s) q_0^{s_1} q_2^{s_2} \cdots q_{2n-2}^{s_n} = (ab+2)^{n-2},$$

$$\sum_{s_1+2s_2+\cdots+ns_n=n} (-1)^{s_1+\cdots+s_n} p_n(s) q_1^{s_1} q_3^{s_2} \cdots q_{2n-1}^{s_n} = -ab(ab+1)^{n-2},$$

$$\sum_{s_1+2s_2+\cdots+ns_n=n} (-1)^{s_1+\cdots+s_n} p_n(s) q_3^{s_1} q_5^{s_2} \cdots q_{2n+1}^{s_n} = -ab,$$

$$\sum_{s_1+2s_2+\cdots+ns_n=n} (-a)^{-s_1-\cdots-s_n} p_n(s) q_4^{s_1} q_6^{s_2} \cdots q_{2n+2}^{s_n} = 0, \qquad n \geq 3,$$

$$\sum_{s_1+2s_2+\cdots+ns_n=n} (-a^2b-2a)^{-s_1-\cdots-s_n} p_n(s) q_6^{s_1} q_8^{s_2} \cdots q_{2n+4}^{s_n} = (ab+2)^{-n},$$

$$\sum_{s_1+2s_2+\cdots+ns_n=n} (-1)^{s_1+\cdots+s_n} (ab+1)^{n-s_1-\cdots-s_n} p_n(s) q_5^{s_1} q_7^{s_2} \cdots q_{2n+3}^{s_n} = -ab,$$

*where the summation is over nonnegative integers satisfying* $s_1+2s_2+\cdots+ns_n = n$ *and* $p_n(s) = \dfrac{(s_1+\cdots+s_n)!}{s_1!\cdots s_n!}$ *is the multinomial coefficient.*

We establish also two formulas expressing numbers $q_i$ with even (odd) subscripts via recurrent determinants of tridiagonal matrices of order $n$.

**Proposition 2.** *For* $n \geq 1$*, the following formulas hold:*

$$q_{2n} = \frac{1}{\prod_{i=1}^{n-2} q_{2i}} \cdot \begin{vmatrix} a & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -aq_3 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -aq_5 & q_2 & q_2 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -aq_{2n-3} & q_{2n-6} & q_{2n-6} \\ 0 & 0 & 0 & 0 & \cdots & 0 & -aq_{2n-1} & q_{2n-4} \end{vmatrix}$$

*and*

$$q_{2n-1}=\frac{1}{\prod_{i=1}^{n-2}q_{2i-1}}\cdot\begin{vmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -bq_2 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -bq_4 & q_1 & q_1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -bq_{2n-4} & q_{2n-7} & q_{2n-7} \\ 0 & 0 & 0 & 0 & \cdots & 0 & -bq_{2n-2} & q_{2n-5} \end{vmatrix}.$$

## References


1. *Alp M., Irmak N., Szalay L.* Two-periodic ternary recurrences and their Binet-formula // Acta Mathematica Universitatis Comenianae. – 2012. – № 81 (2). – P. 227–232.
2. *Edson M., Yayenie O.* A new generalization Fibonacci sequence and extended Binet's formula // Integers. – 2009. – № 9. – P. 639–654.
3. *Koshy T.* Fibonacci and Lucas Numbers with Applications. – N. Y.: John Wiley & Sons, 2001.
4. *Yayenie O.* A note on generalized Fibonacci sequence // Applied Mathematics and Computation. – 2011. – № 217. – P. 5603–5611.
5. *Zatorsky R., Goy T.* Parapermanents of triangular matrices and some general theorems on number sequences // Journal of Integer Sequences. – 2016. – № 19 (2). – Article 16.2.2.



**UOD.517.97**


# On determination of initial function in mixed problem for a second order hyperbolic equation


**H.F.Guliyev, V.N.Nasibzade**

***Baku State University,Sumgait State University,***

***e-mail: hkuliyev@ rambler.ru, nasibzade87@mail.ru***


In the paper we consider an inverse problem of determination of an initial function in a mixed problem for a second order hyperbolic equation. This problem is reduced to an optimal control problem and in the new problem the Frechet differentiability is proved**,** necessary and sufficient condition for optimality is derived,