# Determinants of Hessenberg matrices whose entries are $h(x)$–Fibonacci polynomials

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A large class of polynomials can also be defined by Fibonacci-like recurrence relations such yields Fibonacci numbers. Such polynomials are called Fibonacci polynomials [1]. Nalli and Haukkanen [2] introduced the $h(x)$–Fibonacci polynomials $F_{hn}(x)$ are defined by the recurrence relation

$$F_{h,n+1}(x) = h(x)F_{hn}(x) + F_{h,n-1}(x) \quad (n \geq 1)$$

with initial conditions $F_{h0}(x) = 0$, $F_{h1}(x) = 1$, where $h(x)$ is polynomial with real coefficients.

Let $P_{hn}(x)$, $Q_{hn}(x)$, $R_{hn}(x)$ be an $n \times n$ Hessenberg matrices given for all $n \geq 1$ by

$$P_{hn}(x) = \begin{pmatrix} F_{h1}(x) & 1 & \cdots & 0 & 0 \\ F_{h2}(x) & F_{h1}(x) & \cdots & 0 & 0 \\ \dots & \dots & \ddots & \dots & \dots \\ F_{h,n-1}(x) & F_{h,n-2}(x) & \cdots & F_{h1}(x) & 1 \\ F_{hn}(x) & F_{h,n-1}(x) & \cdots & F_{h2}(x) & F_{h1}(x) \end{pmatrix},$$

$$Q_{hn}(x) = \begin{pmatrix} F_{h1}(x) & 1 & \cdots & 0 & 0 \\ F_{h3}(x) & F_{h1}(x) & \cdots & 0 & 0 \\ \dots & \dots & \ddots & \dots & \dots \\ F_{h,2n-3}(x) & F_{h,2n-5}(x) & \cdots & F_{h1}(x) & 1 \\ F_{h,2n-1}(x) & F_{h,2n-3}(x) & \cdots & F_{h3}(x) & F_{h1}(x) \end{pmatrix},$$

$$R_{hn}(x) = \begin{pmatrix} F_{h2}(x) & 1 & \cdots & 0 & 0 \\ F_{h4}(x) & F_{h2}(x) & \cdots & 0 & 0 \\ \dots & \dots & \ddots & \dots & \dots \\ F_{h,2n-2}(x) & F_{h,2n-4}(x) & \cdots & F_{h2}(x) & 1 \\ F_{h,2n}(x) & F_{h,2n-2}(x) & \cdots & F_{h4}(x) & F_{h2}(x) \end{pmatrix}.$$

**Proposition.** *The following formulas are hold*:

$$\det(P_{hn}(x)) = \sum_{k=0}^{n}\sum_{j=0}^{n}\big(1+(-1)^{n+j}\big)\binom{\frac{n+j}{2}}{j}\binom{j}{n-k}\big(-h(x)\big)^{n-k},$$

$$\det(Q_{hn}(x)) = (-1)^{n-1}h^2(x)\big(h^2(x)+1\big)^{n-2},$$

$$\det(R_{hn}(x)) = h(x)\det(R_{h,n-1}(x)) + \sum_{i=1}^{n-1}(-1)^{n+i}F_{h,2(n-i+1)}(x)\det(R_{h,i-1}(x)).$$

[1] T. Koshy, *Fibonacci and Lucas Numbers with Applications*, John Wiley & Sons, New York, 2001.

[2] A. Nalli, P. Haukkanen, On generalized Fibonacci and Lucas polynomials, *Chaos Solitons Fractals* **43** (2009), 3179–3186.